

Indian Statistical Institute, Bangalore Centre

B.Math (Hons) III Year, Second Semester

Semestral Examination

Comb. and Graph Theory

Time: 3 Hours

April 30, 2012

Instructor: N.S.N.Sastry

Total Mark : 100

Note: Answer all questions. Your answers should be complete and clearly written.

1. a) Define a Hadamard matrix of order n . Show that if a Hadamard matrix of order n exists and $n > 3$, then n is a multiple of 4.
b) If q is a prime power and $q \equiv 3 \pmod{4}$, show that there exists a Hadamard matrix of order $q + 1$. [4+6+10]
2. a) If G is a finite strongly regular graph with parameters (v, k, λ, m) , then show that the complement \bar{G} of G is also a strongly regular graph with parameters $(v, v - k - 1, (v - 2) - (2k - \mu), v - 2k + \lambda)$. [10]
b) Give an example of a permutation group of rank 3. Show that any permutation group of rank 3 on a set of v elements yields a strongly regular graph on v vertices. [4+8]
3. a) Define a linear maximum distance separable code. If C is a maximum distance separable code, show that, C^\perp is also a maximum distance separable code. [4+6]
b) Define a $q - r$ Hamming code. Determine its dimension and words of nonzero minimum weight. [4+4+4]
4. Define the linear code C associated with a projective plane of order 4. Determine its dimension and its words of minimum nonzero weight. [4+6]
b) Show that a projective plane of order 4 contains 6 points, no three on a line. [10]
5. a) Let $\mathbb{D} = (X, \mathbb{B})$ be an extension of a symmetric 2- design and $B \in \mathbb{B}$. Show that $(X \setminus B, \{B' \in \mathbb{B} : B \cap B' = \phi\})$ is a 2- design. What are its parameters? [12]
b) If a projective plane of order n extends, then show that n divides 12. [8]
